**United College of Engineering and Research, Prayagraj**

**Department of Computer Science & Engineering**

**IInd Sessional Examination (2020-21)**

**B.Tech. (IIIrd Semester)**

**Discrete Structures and Theory of Logic**

**Subject Code: KCS-303**

**Time:** 2.00 hours **Max. Marks:** 30

**Note:** There are three sections in this paper. All sections are compulsory.

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| **Question No.** | **Question** | **Marks** | **CO** | **Bloom’s level** |
| **Section-A** | | | | |
| 1 | Define modular lattice. | 10 | 3 | L1 |
| 2 | Define Boolean algebra. | 3 | L1 |
| 3 | Define complemented lattice. | 3 | L1 |
|  | The inclusion of \_\_\_\_\_\_ sets into R = {{1, 2}, {1, 2, 3}, {1, 3, 5}, {1, 2, 4}, {1, 2, 3, 4, 5}} is necessary and sufficient to make R a complete lattice under the partial order defined by set containment. (A) {1}, {2, 4} (B) {1}, {1, 2, 3} (C) {1} (D) {1}, {1, 3}, {1, 2, 3, 4}, {1, 2, 3, 5} | 3 | L2 |
| 5 | A relation R is defined on ordered pairs of integers as follows:-  (x,y)R(u,v) if x < u and y > v. Then R is:   1. Neither a partial order nor an equivalence relation 2. A partial order but not a total order 3. A total order 4. An equivalence relation | 3 | L2 |
| 6 | Let P, Q and R be tree atomic prepositions assertions. Let X denotes (P∧Q) →R and Y denotes (P→R) ∧ (Q→R). Which one of the following is a tautology?  (A) X∨Y (B) X→Y (C) Y→X (D) ˥Y→X | 4 | L2 |
| 7 | Show that the propositions 𝑝→𝑞 𝑎𝑛𝑑 ˥𝑝∨𝑞 are logically equivalent. | 4 | L2 |
| 8 | Write the contra positive of the implication: “if it is Sunday then it is a holiday”. | 4 | L2 |
| 9 | Which one of the following is not equivalent to p ↔q?   1. (˥𝑝∨𝑞) ∧ (𝑝∨˥𝑞) (B) (˥𝑝∨𝑞) ∧ (q→p) 2. (˥𝑝 ∧ 𝑞) ∨ (𝑝 ∧ ˥𝑞) (D) (˥𝑝 ∧ ˥𝑞) ∨ (𝑝 ∧ 𝑞) | 4 | L2 |
| 10 | Identify the correct translation into logical notation of the following assertion.  Some boys in the classes are taller than all the girls  Note: taller(x,y) is true if x is taller than all the girls.   1. (∃x)( boy(x) → (∀y)(girl(y) ∧ taller(x,y))) 2. (∃x)( boy(x) ∧ (∀y)(girl(y) ∧ taller(x,y))) 3. (∃x)( boy(x) → (∀y)(girl(y) → taller(x,y))) 4. (∃x)( boy(x) ∧ (∀y)(girl(y) ∧ taller(x,y))) | 4 | L3 |
| **Section-B** | | | | |
| 1. **Attempt any three.** | | | | |
|  | Prove that a lattice with 5 elements is not a Boolean algebra. | 2 | 3 | L3 |
|  | Show that the following are equivalent in a Boolean algebra  a ≤ b⇔ a∧b' = 0⇔b' ≤ a’ ⇔ a’∨ b = 1 | 2 | 3 | L3 |
|  | Let (L,∨,∧,≤) be a distributive lattice and a, b∈ L . if a ∧ b = a ∧ c and  a ∨ b = a ∨ c then show that b = c. | 2 | 3 | L3 |
|  | Simplify the following Boolean function using three variables maps:  i. f(x,y,z)=Σ(0,1,5,7)  ii. f(x,y,z)=Σ(1,2,3,6,7) | 2 | 3 | L3 |
| 1. **Attempt any three.** | | | | |
|  | Show that ((P ∨Q) ∧˥(˥ Q∨ ˥R)) ∨ (˥P∨ ˥Q) ∨ (˥P∨ ˥R) is a tautology by using equivalences. | 2 | 4 | L3 |
|  | Obtain the principle disjunctive and conjunctive normal forms of the formula  ( p→r) ∧ ( q↔ p) | 2 | 4 | L3 |
|  | Prove the validity of the following argument “if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed”. | 2 | 4 | L3 |
|  | Verify that the given propositions are tautology or not.   1. p ∨˥ (p ∧q) 2. ˥p ∧q | 2 | 4 | L2 |
| **Section-C** | | | | |
| 1. **Attempt any one.** | | | | |
|  | Answer these questions for the poset({3, 5, 9, 15,24, 45}, |).  i. Find the maximal elements. ii. Find the minimal elements.  iii. Is there a greatest element? iv. Is there a least element?  v. Find all upper bounds of {3, 5}.vi. Find the least upper bound of {3, 5}.  vii. Find all lower bounds of {15, 45}. viii. Find the greatest lower bound of {15, 45}, if it exists. | 4 | 3 | L4 |
|  | In a Lattice if a≤ b ≤c , then show that   1. a∨b = b∧c 2. (a∨b)∨(b∧c) = (a∨b) ∧ (a∨c) = b | 4 | 3 | L3 |
| 1. **Attempt any one.** | | | | |
|  | Prove the validity of the following argument:-  If I get the job and work hard then I will get promoted. If I will get promoted, then I will be happy. I will not be happy. Therefore I will not get the job or I will not work hard. | 4 | 4 | L3 |
|  | Rewrite the following arguments using quantifiers, variables and predicate symbols.   1. All birds can fly. 2. Some men are genius. 3. Some numbers are not rational. 4. There is a student who likes mathematics but not geography. | 4 | 4 | L3 |

**Bloom’s taxonomy level**  (1- Remembering, 2. Understanding, 3. Applying, 4. Analyzing, 5. Evaluating, 6. Creating)

**CO** -- Course Outcome